

Multivariate CLT for Lévy processes: convergence rates without moment assumptions

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Classical setting: CLT and its rate of convergence

Let $(X_i)_{i \in \mathbb{N}}$ iid with $\mathbb{E}X_1^2 = 1$, $\mathbb{E}X_1 = 0$, then $S_n = \sum_{i=1}^n X_i$ satisfies

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Question 2 Does the $\sqrt{t}$ -scaling in multivar. CLT yield best rate in $d_{\mathcal{C}}(\mathbf{X}_t/\sqrt{t}, \mathbf{Z}) \rightarrow 0$?

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Theorem 1 (Characterisation of the finite variance of a Lévy process in $\mathbb{R}^d$)

$\mathbf{X}$ Lévy process, $\mathbf{Z}$ standard Gaussian vector in $\mathbb{R}^d$. Then the following are **equivalent**:

- (a) $\mathbb{E}[|\mathbf{X}_1|^2] < \infty$.
- (b) $\exists$ measurable $\mathbf{A} : [1, \infty) \rightarrow \mathbb{R}^d$ and $\mathbf{B} : [1, \infty) \rightarrow \mathbb{R}^{d \times d}$, such that $\mathbf{B}(t)$ is invertible for large t , smallest eigenvalue $\lambda_{\min}(\mathbf{B}(t)^\top \mathbf{B}(t)) \rightarrow \infty$ and $\mathbf{B}(t)^{-1} \mathbf{B}(f(t)) \rightarrow \mathbf{I}_d$ holds for any non-decreasing functions f with $f(t)/t \rightarrow 1$ as $t \rightarrow \infty$ and

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If $\mathbb{E}[|\mathbf{X}_1|^2] < \infty$, set $\mathbf{A}_c(t) := t\mathbb{E}[\mathbf{X}_1]$ & $\mathbf{B}_c(t) := \sqrt{t}\mathbf{\Delta}(t)$, where $\mathbf{\Delta}(t) := \sqrt{\mathbf{\Sigma}(t)}$ and

$$\mathbf{\Sigma}(t) := \mathbf{\Sigma} + \int_{\{|\mathbf{v}| < \kappa\sqrt{t}\}} \mathbf{v}\mathbf{v}^\top \nu(d\mathbf{v}) \rightarrow \mathbf{\Sigma} + \int_{\mathbb{R}^d \setminus \{\mathbf{0}\}} \mathbf{v}\mathbf{v}^\top \nu(d\mathbf{v}) = \mathbb{E}[(\mathbf{X}_1 - \mathbb{E}\mathbf{X}_1)(\mathbf{X}_1 - \mathbb{E}\mathbf{X}_1)^\top] =: \boldsymbol{\sigma}^2.$$

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Theorem 2 (Characterisation of $(2 + \log)$ -moments)

Let $\mathbf{Z}$ be standard Gaussian in $\mathbb{R}^d$ and $\mathbf{X}$ Lévy process with second moment $\mathbb{E}[|\mathbf{X}_1|^2] < \infty$. Denote the covariance matrix of $\mathbf{X}_1$ by $\boldsymbol{\sigma}^2 = \mathbb{E}[(\mathbf{X}_1 - \mathbb{E}\mathbf{X}_1)(\mathbf{X}_1 - \mathbb{E}\mathbf{X}_1)^\top]$. Then:

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- This explains why polynomial Berry-Esseen type bounds under $(2 + \delta)$ -moment assumption **have to** deteriorate arbitrarily as $\delta \downarrow 0$.

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Hence: $\mathbf{X}$ in DoNNA $\implies \mathbb{E}[|\mathbf{X}_1|^2] = \infty = \int_{\{|\mathbf{v}| > 1\}} |\mathbf{v}|^2 \nu(d\mathbf{v})$ & $\limsup_{t \rightarrow \infty} \frac{\text{tr}(\mathbf{B}(t))}{\sqrt{t}} = \infty$.

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Corollary 3

If $\mathbf{X}$ in DoNNA, then for any $\mathbf{A} : [1, \infty) \rightarrow \mathbb{R}^d$ and $\mathbf{B} : [1, \infty) \rightarrow \mathbb{R}^{d \times d}$, s.t. $\mathbf{B}(t)$ invertible for large t , $\lambda_{\min}(\mathbf{B}(t)^\top \mathbf{B}(t)) \rightarrow \infty$ and $\mathbf{B}(t)^{-1} \mathbf{B}(f(t)) \rightarrow \mathbf{I}_d$ for any non-decreasing functions f with $f(t)/t \rightarrow 1$ as $t \rightarrow \infty$ we have

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Hard bound: since $d_{\mathcal{H}} \leq d_{\mathcal{C}}$, convex distance $d_{\mathcal{C}}(\mathbf{B}(t)^{-1}(\mathbf{X}_t - t\mathbb{E}[\mathbf{X}_1]), \mathbf{Z})$ cannot be upper bounded by a function that is integrable with respect to $t^{-1}dt$ on $[1, \infty)$, cf. [GCKBM25].

Elements of the proofs of Theorems 1 and 2

Thm 1 (implication: (a) : $\mathbb{E}|\mathbf{X}_1|^2 < \infty \implies$ (b) : $t \mapsto t^{-1}d_{\mathcal{C}}(\mathbf{X}_t - \mathbf{A}(t), \mathbf{B}(t)\mathbf{Z}) \in L^1_{\text{loc}}(+\infty)$):

- multivariate Berry-Esseen theorem applied to the Lévy process with truncated jumps
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- generalisation of a limit theorem [AR01, Lem. 3.1] in the small-time regime

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Thm 2 $\mathbb{E}[|\mathbf{X}_1|^2 \max\{0, \log(|\mathbf{X}_1|)\}] < \infty \iff t \mapsto t^{-1}d_{\mathcal{C}}((\mathbf{X}_t - t\mathbb{E}\mathbf{X}_1)/\sqrt{t}, \boldsymbol{\sigma}\mathbf{Z}) \in L^1_{\text{loc}}(+\infty)$

- since $\mathbb{E}[|\mathbf{X}_1|^2] < \infty$, we have $\boldsymbol{\Delta}(t) \rightarrow \boldsymbol{\sigma}$ as $t \rightarrow \infty$. By triangle inequality

$$d_{\mathcal{C}}((\mathbf{X}_t - t\mathbb{E}\mathbf{X}_1)/\sqrt{t}, \boldsymbol{\sigma}\mathbf{Z}) \leq d_{\mathcal{C}}((\mathbf{X}_t - t\mathbb{E}\mathbf{X}_1)/\sqrt{t}, \boldsymbol{\Delta}(t)\mathbf{Z}) + d_{\mathcal{C}}(\boldsymbol{\Delta}(t)\mathbf{Z}, \boldsymbol{\sigma}\mathbf{Z})$$

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- argument showing that convergence in distribution **cannot** hold if $\mathbb{E}|\mathbf{X}_1|^2 = \infty$ (using ideas from classical but little known work of Heyde [Hey69, Hey73] for one-dimensional random walks).
- generalisation to continuous time poses significant technical difficulties (e.g. summability in discrete time [Hey69, Hey73] implies Kolmogorov distance $\rightarrow 0$; in continuous time integrability does not!)

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Thank you for your attention!!

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